
Controlling the Shape of a Muscular Hydrostat: A tongue or tentacle

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Summary. A mathematical model of the human tongue, developed as part of a larger project to understand its movement, use in speech, and control, is described. The model has the mathematical form of an incompressible solid with a nonlinear, partially controllable, stress/strain relationship. The rationale for such a model, an analytic solution for a simplified version of the model, and indications of future work are included.

1 Introduction

The human tongue is an important component of the system that produces intelligible speech. To understand speech production, it is necessary to understand the dynamics and control of the tongue. There are several difficulties. First, the tongue is enclosed in the mouth during speech, greatly complicating the problem of obtaining accurate experimental data about its movements. Second, the tongue is one of a small collection of biomechanical systems known as muscular hydrostats. Such systems—other examples are the elephant’s trunk and the squid’s tentacles—are composed almost exclusively of muscle and do not include any rigid structure for the muscles to act upon.

Muscles can only shorten as a result of their own activation. To lengthen, a muscle must be acted upon by some force external to itself. In the case of the tongue, or any other muscular hydrostat, this force is created by the

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interaction of other muscles within the tongue with the incompressibility of the tongue. Thus, the control of the tongue depends on exploiting the dynamical interaction between the contraction of individual muscles and the incompressibility. To understand this quantitatively, it is necessary to describe the tongue mathematically as an incompressible elastic structure that is controlled by temporally varying the stress/strain relationship within subregions of the overall structure. This article describes such a mathematical model of the tongue and the squid tentacle.

Before describing our model, we present a brief survey of previous attempts to model the tongue mathematically. We follow this with a demonstration that a linear model of the tongue that enforces incompressibility by means of a Poisson's ratio close to $1/2$ does not both replicate reasonable movements of the tongue and satisfy the incompressibility constraint. We then give a new mathematical model of the tongue. The model is designed to handle large deformations because experimental data shows that the tongue normally undergoes large deformations. A crucial component of the model is a mathematical model of mammalian striated muscle, which is described in the next section. We conclude with a brief indication of the goals of this research and of the next steps towards achieving these goals.

2 Background

The first attempt to create a mathematical model of the human tongue is generally acknowledged to be by Perkell in 1969 [11]. Perkell used interconnected masses and springs to create a two-dimensional mathematical model of the mid-sagittal plane of the tongue. Kiritani et al.[9] substantially improved upon Perkell's work by creating a three-dimensional finite element model. Their model also included simple models of the lips and vocal tract and of tongue contact. Improved models of the tongue were later developed by Hashimoto and Suga in [7] and Sanguinetti et al. in 1997-1998 [12, 13]. At about the same time, Payan and Perrier [10] developed a two-dimensional model of the midsagittal plane of the tongue that included some geometrical nonlinearities. Their model was based on the somewhat controversial equilibrium point model of muscle. Dang and Honda [3] have continued and extended the development of two-dimensional models by including 2cm of transverse thickness in their essentially two-dimensional model. Their model was based on measurements from volumetric MRI and the tongue, jaw, hyoid bone, and vocal tract wall. However, their model was based on a mass-spring network that did not fully account for continuum effects. More recently, they have improved their model so that it includes the effects of Poisson's ratio and produces a "semicontinuum model" tissue model [4].

A somewhat different approach to the mathematical modelling of the tongue has been the work of Wilhelms-Tricarico [15]. Wilhelms-Tricarico's approach to modelling has been to create very detailed models that attempt

to include as much as is possible of the known elements of the tongue. The models begin with a finite element grid that is tailored to the shapes of individual muscles within the tongue. A considerable amount of nonlinearity is included. For example, the model includes the nonlinear force/length and force/velocity relations for mammalian muscle developed by Zajac [16].

The reported aspects of virtually all of this work do not address the mathematical issues associated with the models. In particular, the models are almost always a single finite element approximation to the physical problem of describing the biomechanics of the tongue. Questions such as whether a refined set of elements would improve the approximation cannot be addressed. Furthermore, in almost all of these models, the finite element mesh is manually created so as to conform to the placement of muscles in the model. The resulting mesh may not satisfy the regularity criteria needed for good finite element computations. A notable exception is the recently developed finite element model of Gerard et al. [6] in which the regularity issue is addressed. However, it is still fair to say that the mathematical issues associated with models of the tongue have mostly been ignored in the published literature.

3 Volume Changes

In many finite element models of the tongue, a value of Poisson's ratio close to one-half is used to model the incompressibility constraint. A common measurement of how well incompressibility is enforced in this type of model is to compute a relative change in the overall volume, e.g.,

$$\text{Relative change in global volume} = \frac{\text{Initial global volume} - \text{Deformed global volume}}{\text{Initial global volume}} * 100$$

If the relative change is small in absolute value then enforcement of incompressibility in the model is considered to be acceptable.

However, incompressibility is a *local* phenomenon, and global volume preservation does not imply local volume preservation. We propose a new method for measuring incompressibility and present two examples where incompressibility appears to be accurately modelled when using the common global measurement, but our new method indicates significant local changes in volume.

Our new method for measuring incompressibility is to compute the relative change in volume at the finite element (FE) level; that is, for each finite element, compute

$$\text{Relative change in each FE volume} = \frac{\text{Initial FE volume} - \text{Deformed FE volume}}{\text{Initial FE volume}} * 100$$

Since there may be many finite elements, one may wish to save these in an array. Using this measurement, we can determine the maximum and minimum

percentage of relative changes in local volume. In particular, we can graph the locations where these changes are above or below a threshold amount, and our model is failing to enforce the incompressibility constraint. We also compute a sum of the volume differences; e.g.,

$$\text{Sum of FE volume difference} = \sum_{FEs} |\text{Initial FE volume} - \text{Deformed FE volume}|$$

as an additional measurement tool.

Using our three-dimensional rectangular prism tongue model, we activated the first section of superior longitudinalis (SL1) and genioglossus (GG1) as shown in Figure 1. The relative change in global volume is .0488%; that is, the

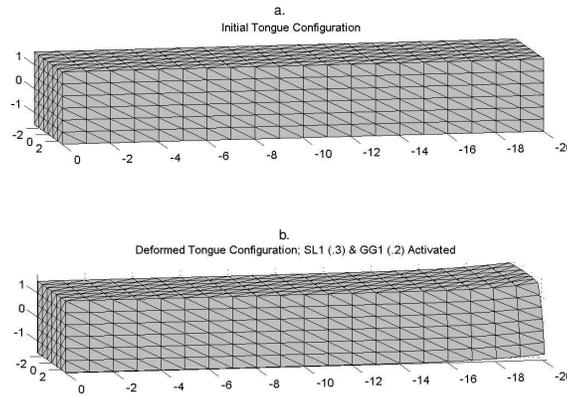


Fig. 1. a. Initial configuration with volume of 360 pixels². b. Deformed tongue after activating SL1 (.3) and GG1 (.2) with volume of 360.1759 pixels².

overall volume increased slightly. Normally, this measurement would indicate that the enforcement of incompressibility is good. Using our new method, we find that the sum of the absolute values of volume differences in the finite elements is .3534, and the relative local volume changes are between 1.4645% and -1.0788% with the negative sign indicating a volume decrease. These numbers show there are areas where the incompressibility constraint is not being enforced.

In our second example, we activated the first section of genioglossus (GG1) in our two-dimensional tongue model. The initial and deformed configurations are almost identical as shown in Figure 2. The relative global volume change is .028287%, which indicates a reasonably good enforcement of incompressibility. However, the sum of the absolute values of finite element volume differences is 1.2350, and the local relative volume changes are between $-.1422\%$ and

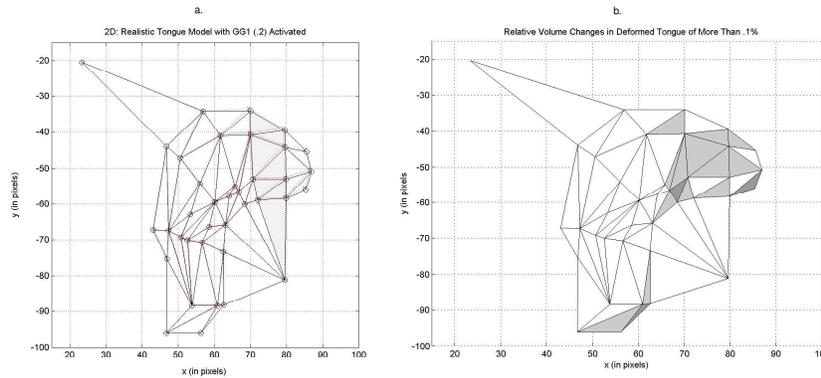


Fig. 2. a. The initial (dashed lines) and deformed (solid black lines) configurations for tongue with GG1 (.2) activated. GG1 lies in the shaded area. b. The shaded areas indicate local relative volume changes above .10% in absolute value. The darker areas indicate volume increases while the lighter areas indicate reductions in volume.

.4795%. Figure 2 shows the locations where the incompressibility constraint is not satisfied.

The above examples were chosen because the activation parameters appear to create small deformations that satisfy the incompressibility constraint. However, on closer examination, there are regions within the deformed tongue where the incompressibility constraint has not been satisfied.

In the above examples, we were careful to keep our activation parameters small because the model used is a small deformation model. We also had to activate only one or two muscle sections at a time because activating more muscle sections even with very small parameters can result in large movements. This result supports our belief that a linear small deformation model cannot adequately model tongue movement. It is also interesting to note that both our new methods and the common technique for measuring incompressibility indicate the incompressibility constraint is not satisfied for these large deformations. This indicates the incompressibility constraint needs to be explicitly included in the finite element model. We propose using a mixed formulation [8] with both displacement and pressure finite elements to approximate a solution that satisfies the incompressibility constraint.

4 Tongue model

The human tongue is incompressible and normally undergoes large deformations. A mathematical model that accurately describes an incompressible elastic structure that undergoes large deformations must be nonlinear. There are many modelling and mathematical issues that simply do not arise in the linear theory but are fundamental to the nonlinear theory. For example, the

distinction between the material and spatial (Lagrangian and Eulerian) formulations of the dynamics is largely irrelevant for small deformations, the case described by linear models. In the case of large deformations, this distinction is very important. In fact, there are strong arguments for using the material formulation for the tongue model (Antman [1] p.ix). However, we do note that the spatial formulation can be useful for some inverse problems ([1] p. 451).

Similarly, there are many choices for the strain tensor. All of them give equivalent results in the linear case but this is not so in the nonlinear case. The main driver of our choice of strain tensor was the principle of frame indifference. This says that the stress/strain relationship of a material cannot change when the material undergoes rigid motions ([1] p. 420 et seq.). This principle implies that the stress/strain relationship should have a particular form that depends primarily on the right Cauchy-Green deformation tensor $\underline{C}(\underline{z}, t)$.

With the above-mentioned conditions in mind, our tongue model takes the following form. Let \underline{z} denote a material point of the tongue belonging to a set of points, \mathbf{B} , in three dimensional space. Let $\underline{p}(\underline{z}, t)$ denote the position of material point $\underline{z} \in \mathbf{B}$ at time t . Then,

$$\underline{F} = \underline{p}_{\underline{z}} \quad (1)$$

$$\underline{C} = \underline{F}^T \underline{F} \quad (2)$$

where \underline{C} is the previously mentioned right Cauchy-Green deformation tensor, \underline{F} is the deformation gradient, and the subscript indicates the partial derivative. Then, the fundamental dynamical equation has the form.

$$\nabla \cdot \underline{T}^T + \underline{f} = \underline{p}_{tt} \quad (3)$$

where \underline{T} is the first Piola-Kirchoff stress tensor, \underline{f} is the body force due to gravity, the double subscript denotes the second partial derivative with respect to t , and we have assumed the mass density of the tongue is a constant that we normalize to one. The actual modelling of the tongue consists primarily of specifying \underline{T} in terms of \underline{F} and \underline{C} .

Because the tongue is nearly incompressible we describe it by an incompressible model. This adds the condition

$$\det \underline{C} = 1 \quad (4)$$

Incompressibility also imposes an important condition on the form of \underline{T} . Precisely,

$$\underline{T}(\underline{z}, t) = \underline{F}(\underline{z}, t)(-p\underline{C}^{-1}(\underline{z}, t) + \hat{\underline{S}}_A(\underline{C}(\underline{z}, t), p, \underline{z})) \quad (5)$$

Note that p without the underline denotes the pressure within the tongue, which is generally a function of \underline{z} and t . The definition of $\hat{\underline{S}}_A$ is really part of the muscle model and will be given below. There is a further condition on

$\hat{\underline{S}}_A$ that we give below. It is an additional component of the incompressibility constraint,

$$\text{tr}(\hat{\underline{S}}_A \underline{C}^{-1}) = 0 \quad (6)$$

Equations (3-6) constitute an abstract mathematical description of the tongue. At this level of abstraction, the equations characterize any incompressible elastic structure in either two or three dimensions. In addition, a model for a compressible elastic structure consists of Equations (3) and (5) with $p = 0$. It is the muscle model and the specific choice of \mathbf{B} that makes these equations describe the tongue.

5 Muscle model

The new muscle model was driven by three mathematical issues in addition to the obvious requirement that it fit the known physiological facts about mammalian muscle. The model had to be convenient to use, satisfy the frame indifference principle described earlier, and had to satisfy reasonable conditions for existence and uniqueness of solutions when incorporated into the overall tongue model. The resulting model is in two parts, the geometric description and the stress/strain characteristics.

a. Geometric description: Each muscle occupies a region of space, a subset of the material points of the tongue. This set changes as the muscle moves. We identify the muscle by the set of points, $\mathbf{M} \subset \mathbf{B}$, that it occupies in its reference configuration. A muscle consists of fibers. We denote these fibers by curves defined on \mathbf{M} in its reference configuration. There is a unique fiber at every point $\underline{z} \in \mathbf{M}$. We define the fibers as a family of offset curves [2]. That is,

(i) A backbone curve, $\underline{c}(s)$, where s is the arc length along the backbone curve. For muscle fibers, s can always be chosen to be a coordinate direction, z_1 for instance. (ii) At each point of the backbone curve, which we can identify by s , there is a unit tangent vector, denoted by $\underline{\tau}(s)$ and, in three dimensions, a plane normal to the curve. Choose a pair of orthonormal basis vectors for this perpendicular plane, say $\underline{\eta}_1(s)$ and $\underline{\eta}_2(s)$. Then every other muscle fiber can be represented by an offset curve, $\underline{q}(s)$, defined by

$$\underline{q}(s) = z_{01}\underline{\eta}_1(s) + z_{02}\underline{\eta}_2(s) \quad (7)$$

for all z_{01} and z_{02} such that $\underline{q}(s) \in \mathbf{M}$. This form assumes the muscle fibers are all parallel in the reference configuration. This is not true for Genioglossus and Verticalis in the tongue. There is a simple generalization of Equation (7) that allows us to represent such fan-shaped muscles. Let z_{0i} , $i = 1, 2$ depend on s .

b. Stress/strain description: Our basic assumption is that the stress produced by a muscle is aligned with the muscle fiber. Stress is described by a tensor so the previous assumption describes one of the principle axes of the stress

tensor. We also assume that the stress/strain relationship in the plane normal to the muscle fiber is entirely passive elastic and independent of direction within the plane. Because this passive elasticity is entirely due to collagen fibers, which have no shear resistance, we assume there is no shear component. These assumptions imply that the stress tensor for a muscle has the form

$$\underline{S}(\underline{z}, t) = \text{diag}(S_A(\underline{z}, t), S_P(\underline{z}, t), S_P(\underline{z}, t)) \quad (8)$$

where $S(\underline{z}, t)$ is the second Piola-Kirchoff stress tensor. Note that the isolated muscle fibers would also have stress resulting from the incompressibility of the muscle. This is accounted for in the overall tongue model as a component of the new tongue model (see Eqn. (4)). We emphasize the alignment of $\underline{S}(\underline{z}, t)$ by noting that

$$\underline{S}(\underline{z}, t)\underline{\tau}(\underline{z}, t) = S_A(\underline{z}, t)\underline{\tau}(\underline{z}, t) \quad (9)$$

and

$$\underline{S}(\underline{z}, t)\underline{\eta}_i(\underline{z}, t) = S_P(\underline{z}, t)\underline{\eta}_i(\underline{z}, t) \quad (10)$$

for $i = 1, 2$. The stress actually depends on the deformation. The requirement that this dependence be frame indifferent implies that $S_A(\underline{z}, t)$ and $S_P(\underline{z}, t)$ can only depend on $\underline{C}(\underline{z}, t)$, its derivatives, \underline{z} , and t . The next step is to give explicit expressions for $S_A(\underline{z}, t)$ and $S_P(\underline{z}, t)$. The passive elasticity is assumed to be given by

$$S_P(\underline{z}, t) = -\hat{S}_P(\sqrt{\underline{\eta}^T(\underline{z}, t)\underline{C}(\underline{z}, t)\underline{\eta}(\underline{z}, t)}) \quad (11)$$

for any unit vector, $\underline{\eta}(\underline{z}, t)$ perpendicular to $\underline{\tau}(\underline{z}, t)$, the tangent vector to the fiber. Note that we are assuming that any viscosity in the muscle can be neglected. This is an assumption that is very commonly made and it is certainly true for static problems and almost certainly true for simulations. It may not be true for questions related to stability. The active stress, that is, the stress produced by muscle activation, is assumed to be given by

$$S_A(\underline{z}, k(t), t) = -\hat{S}_A(\sqrt{\underline{\tau}^T(\underline{z}, t)\underline{C}(\underline{z}, t)\underline{\tau}(\underline{z}, t)}, \sqrt{\underline{\tau}^T(\underline{z}, t)(\partial\underline{C}(\underline{z}, t)/\partial t)\underline{\tau}(\underline{z}, t)}, \underline{z}, k(t)) \quad (12)$$

where $k(t)$ is the control, $0 \leq k(t) \leq 1$ i.e., the muscle activation.

Notice that \hat{S}_P is a scalar function of a scalar argument. It has the functional form

$$\hat{S}_P(x) = \begin{cases} 0, & \text{if } x \leq 1; \\ \text{monotonic increasing to infinity} & \text{if } x > 1 \end{cases} \quad (13)$$

\hat{S}_A is a scalar function of three scalar arguments. It has the functional form

$$\hat{S}_A(x, y, k) = kf_l(x)f_v(y) + \hat{S}_P(x)$$

Here, f_l corresponds to the force-length curve of the Hill model and f_v corresponds to the force velocity curve. Again, we impose the condition that $\hat{S}_A(x, y, k)$ is monotonic increasing in its first argument regardless of the other arguments. This is a common assumption in using Hill's model [16] because the system can be unstable otherwise.

6 Muscle placement

The mathematical model of the tongue is not complete without a geometric description of the tongue. In [14], Takemoto developed a three-dimensional schemata of the musculature based on anatomical observations. In his work, he identified five distinct strata of muscles and provided a description of the muscle fiber locations within each stratum. Our geometric description is based primarily on his work.

Some key issues in developing a geometric description of the tongue are determining what muscles to include, identifying their location within the tongue, and defining their initial direction or backbone curve. The tongue has a complex musculature and is comprised of many muscles, both intrinsic and extrinsic. Intrinsic muscles are muscles that originate and terminate in the soft tissue within body of the tongue while extrinsic muscles originate on structures outside the body, such as the jaw, insert into the tongue, and terminate within the tongue body.

In his paper, Takemoto identified four intrinsic and four extrinsic muscles. Our geometrical description includes the four intrinsic muscles: superior longitudinalis (SL), inferior longitudinalis (IL), transverse (T), and verticalis (V). These muscles compress the tongue in three directions: front-to-back (SL, IL), top-to-bottom (V), and side-to-side (T). The longitudinal muscles also raise and lower the tip. We also include two extrinsic muscles: genioglossus (GG) and hyoglossus (HG) and are in the process of incorporating a third extrinsic muscle, styloglossus (SG), into the model, but this work is not completed. For simplicity, we have chosen not to include the fourth muscle, palatoglossus. The extrinsic muscles pull the tongue toward the muscle's origin or attachment location and deform the tongue according to their insertions. For example, genioglossus is a fan shaped muscle that originates at the jaw. Its fibers enter along the center underbody of the tongue and extend upward terminating along the entire upper tongue surface at midline and a few millimeters on either side. Genioglossus pulls local regions toward the jaw forming midline depressions and grooves.

A muscle is comprised of many individual muscle fibers, which have a cylindrical shape. Individual muscle fibers may extend from origin to insertion or may be much shorter. These shorter fibers are either aligned in parallel, i.e., running end-to-end and side-by-side, or may be in-series with short muscle fibers that overlap. The space in between the fibers contains an intricate collagen network or mesh. While extrinsic muscles have a clearly defined starting

point, the termination of these fibers as well as the origination and termination of intrinsic muscle fibers are not as well defined. Takemoto's dissections[14] clearly indicate that the number of fibers from each muscle changes from slice to slice. In addition, Takemoto identified two alternating laminae of muscle fibers in the inner region or body of the tongue. These laminae were each approximately one fiber thick. One of the lamina is composed of genioglossus and verticalis muscle fibers while the other contains transverse fibers. For simplicity, we assume each muscle has a clearly defined boundary and that each muscle is homogeneous within that boundary, i.e. muscle fibers are evenly distributed. In particular, we assume all muscle fibers are aligned in parallel. We further assume that an increase in the activation of a muscular region results in an evenly distributed recruitment of fibers throughout the muscle location. We have currently made no attempt to model the alternating lamina but plan to study the effects of this muscle structure in future geometric descriptions after we have identified the limitations of our simplifying assumptions.

We present a brief description of the muscle locations within our tongue geometry. In order to define the tongue and individual muscle locations, we use a coarse finite element structure. In the future, these definitions will be independent of the finite element structure. SL is located within a thin region extending along the top of the tongue from the tip to the base. Ferrari et al.[5] hypothesized individual superior longitudinalis fibers, which have an in-series configuration, can have differing activation levels along their length allowing for independent movement regionally. In order to test this hypothesis, we divided SL into six sections along its length, each with its own activation parameter. We are, of course, assuming that SL has such clear boundaries along its length where independent muscle activation occurs. The shaded areas in Figure 3 show the region containing SL and our divisions. IL lies symmetrically on either side of the midsagittal plane and extends along the bottom of the tongue body from the tip to the base as shown in Figure 3.

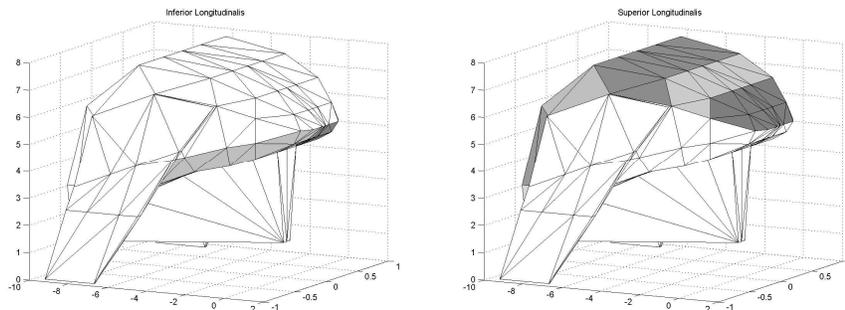


Fig. 3. The location of Inferior Longitudinalis is shown at the left and Superior Longitudinalis at the right. Shading indicates independently controllable sections of the muscle.

T and V lie in the body of the tongue above IL. V lies symmetrically on either side of a region containing the midsagittal plane while T crosses this midsagittal region. We have divided T and V along their widths as shown in Figure 4 to allow for independent activation. Activation of the full muscle is achieved by choosing the same activation parameters for all of the shaded regions.

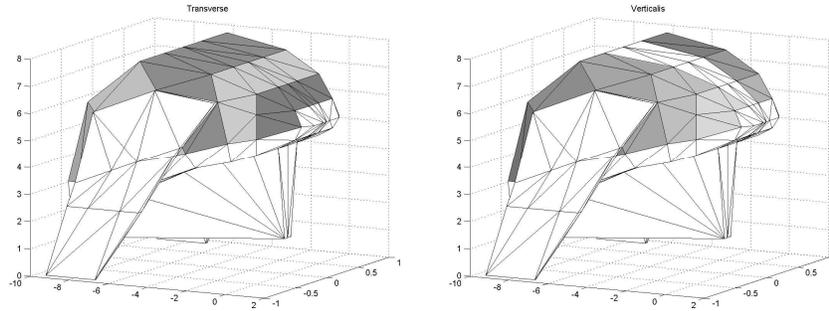


Fig. 4. The location of Transversalis is shown at the left and Verticalis at the right. Shading indicates independently controllable sections of the muscle.

As stated earlier, GG is a fan shaped muscle that is located in the mid-sagittal plane and extends a few millimeters on either side. Its fibers originate at the jaw and extend up into the region containing SL. We have divided GG into five regions along its width to allow for independent activation as shown in Figure 5. The last muscle in our current model, HG, originates at the hyoid bone, extends upward on both sides of the tongue, and enters the tongue laterally. The anterior fibers run upward and then forward remaining

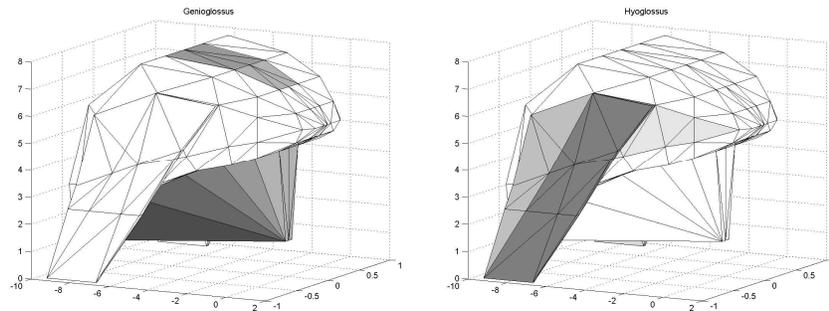


Fig. 5. The location of Genioglossus is shown at the left and Hyoglossus at the right. Shading indicates independently controllable sections of the muscle.

below SL. The medial and posterior fibers run upward and then join with T fibers. We have separated this muscle into three sections, one consisting of the medial and posterior fibers, one with the upward portion of the anterior fibers and the last with the forward portion of the anterior fibers as shown in Figure 5. We are in the process of adding SG to our model. SG originates at the styloid process of the temporal bone, extends downward and forward partially intersecting with HG before entering the tongue. Once SG enters the tongue, posterior fibers run downward toward the base, middle fibers merge with T, and anterior fibers run forward along the sides of the tongue between SL and IL.

7 Example

To illustrate the model and its application, consider the problem of elongating a two-dimensional tentacle. We assume the tentacle has two muscle fibers through each material point. One fiber is vertical and the other horizontal. We model incompressibility in two dimensions by requiring every infinitesimal area to be constant, regardless of muscle activation. The basic idea is illustrated in Figure 6 where the initial position is shown dotted and the desired final position solid. We ignore gravity for simplicity.

Is it possible to control the muscle activation of this tentacle in such a way as to cause it to move from its initial position to its final position while maintaining its rectangular shape at all intermediate stages?

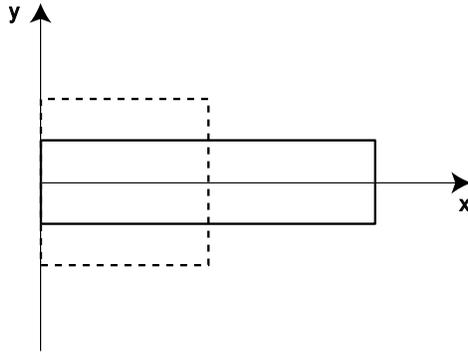


Fig. 6. A simple two-dimensional elongation of an abstract tentacle. Contracting the vertical muscles should cause the height to decrease and the length to increase.

To answer this question, denote material points of the tentacle by $\underline{z} = [x \ y]^T$. Let $\underline{p}(\underline{z}, t)$ denote the position of material point $\underline{z} \in \{-1 \leq x \leq 1, 0 \leq y \leq 1\}$ at time t . Then,

$$\underline{F} = \underline{p}_{\underline{z}} \quad (14)$$

$$\underline{C} = \underline{F}^T \underline{F} \quad (15)$$

where \underline{C} is the previously mentioned right Cauchy-Green deformation tensor and \underline{F} is the deformation gradient and the subscript indicates the partial derivative. A precise characterization of the desired movement of the tentacle is

$$\underline{p}(\underline{z}, t) = [\alpha^{-1}(t)x \quad \alpha(t)y]^T \quad (16)$$

This makes

$$\underline{F}(\underline{z}, t) = \begin{bmatrix} \alpha^{-1}(t) & 0 \\ 0 & \alpha(t) \end{bmatrix}.$$

Then,

$$\underline{C}(\underline{z}, t) = \begin{bmatrix} \alpha^{-2}(t) & 0 \\ 0 & \alpha^2(t) \end{bmatrix}. \quad (17)$$

The incompressibility constraint is automatically satisfied by this $\underline{C}(\underline{z}, t)$. It is necessary to account for the effect of the constraint on the stresses. Physically, the constraint supplies whatever stress is required to insure that the constraint is satisfied. For a given \underline{z} , the incompressibility constraint (4) defines a two dimensional manifold in the three-dimensional inner product space of symmetric 2×2 matrices. The inner product of two such matrices, say \underline{A} and \underline{B} is just $tr(\underline{A}\underline{B})$. An orthonormal basis for this space, aligned with the tangent plane to the manifold, consists of the three matrices, $\underline{B}_1 = \underline{C}^{-1}/\sqrt{\alpha^4 + \alpha^{-4}}$,

$$\underline{B}_2 = \begin{bmatrix} \alpha^{-2} & 0 \\ 0 & -\alpha^2 \end{bmatrix} / \sqrt{\alpha^4 + \alpha^{-4}} \quad (18)$$

and

$$\underline{B}_3 = \begin{bmatrix} 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 \end{bmatrix}$$

We will enforce the constraint (6) by projecting the stress tensor onto the tangent plane of the manifold.

The stress tensor is aligned with the muscle fibers. In this example, the vertical muscle fibers stay vertical and straight throughout the movement and the horizontal muscle fibers stay horizontal. A natural assumption is that only the vertical fibers are activated so as to decrease the height with minimum resistance from other forces. Activating the horizontal fibers would only increase the needed stress. With these assumptions, the stress tensor becomes

$$\underline{S}(\underline{z}, k(t), t) = \begin{bmatrix} -(k(t)f_l(\alpha^{-1}(t))f_v(d\alpha^{-1}(t)/dt) - \hat{S}_P(\alpha^{-1}(t))) & 0 \\ 0 & -\hat{S}_P(\alpha(t)) \end{bmatrix} \quad (19)$$

where $\hat{S}_P(\alpha)$ is defined in (13).

Taking the projection of $\underline{S}(\underline{z}, k(t))$ onto \underline{B}_2 gives

$$\hat{S}_A(\underline{C}(\underline{z}, t), p, \underline{z}, k(t)) = \text{tr}(\underline{S}(\underline{z}, k(t))\underline{B}_2)\underline{B}_2. \quad (20)$$

Finally, Equation(5) becomes, explicitly,

$$\begin{aligned} \underline{T}(\underline{z}, k(t), t) = & \left(\frac{1}{\sqrt{\alpha^4 + \alpha^{-4}}} \right) \left(\begin{bmatrix} -p\alpha & 0 \\ 0 & -p\alpha^{-1} \end{bmatrix} \right. \\ & \left. + \frac{\alpha^{-2}(k(t)f_l(\alpha^{-1}(t))f_v(d\alpha^{-1}(t)/dt) + \hat{S}_P(\alpha^{-1}(t))) - \alpha^2\hat{S}_P(\alpha(t))}{\sqrt{\alpha^4 + \alpha^{-4}}} \begin{bmatrix} \alpha^{-3} & 0 \\ 0 & -\alpha^3 \end{bmatrix} \right) \end{aligned} \quad (21)$$

Note that p is given explicitly by the formula

$$p(\underline{z}, t) = -\text{tr}(\underline{S}(\underline{z}, k(t), t)\underline{B}_1(\underline{z}, t))$$

Inspection of the formula for $\underline{T}(\underline{z}, k(t), t)$ shows that there is no dependence on \underline{z} . Hence, $\nabla \cdot \underline{T}^T(\underline{z}, k(t), t)$ must be 0. Thus, the hypothesized motion of the tentacle is impossible if there is only a single open-loop control of vertical muscle fibers. This would typically be true of muscles in the limbs. We believe it is unlikely to be true for tongue and tentacle muscles.

Suppose we allow the control of the vertical muscle fibers to depend on both the fiber and the position along its length. That is, the control is now $k(t, x, y)$. Choosing $k(t, x, y) = k(t)(x^2 + y^2)$ and substituting this into the equations above reduces the dynamics to a single nonlinear first-order differential equation in $\alpha(t)$ and $k(t)$. We are currently investigating whether this equation is solvable for some choice of $k(t)$.

We have so far said nothing about boundary conditions. The most natural boundary condition is that the component of stress perpendicular to the three edges of the tentacle other than the one on the y -axis be zero. We allow arbitrary vertical motion on the y -axis but zero horizontal motion. A more standard way to achieve the same result is to regard the tentacle as the right half of an object that is symmetric about the y -axis and has zero normal stress on all its boundaries. This leads to a pair of equations that are linear in p and k . Evaluation of the determinant shows the solution is unique for $\alpha > 1$ although the required k might not lie between 0 and 1. If we allow cocontraction, i.e., both sets of muscles to be active, then we have two linear equations in three unknowns and many solutions.

8 Further research

The main purpose of this research is to determine how the activity of the muscles of the tongue controls its movements. Our collaborators have been using tagged cine-MRI and ultrasound to provide the motions of individual points within the tongue as well as lines on its surface. The data we already have

is in two spatial dimensions. We, and they, expect to have three-dimensional data shortly. Given this data, we have an obvious inverse problem—given the movement of points within the tongue, compute the muscle activations that produced this movement. One important biological issue is that the extent to which individual muscle fibers can be independently controlled is unknown. In the muscles of the limbs, for example, there is normally no reason to control the shape; only the force needs to be controlled. Thus, fibers are recruited for a task in strict order of their size and force producing capabilities. Much more complicated recruitment strategies might be needed in the tongue, where shape must be controlled. Moreover, there is no reason to separately control different fibers that are connected in series. In the tongue, subtle control of the shape could require separate control of series connected fibers. Mathematical questions, such as whether a solution to the inverse problem exists and is unique can provide great insight into these, and other, issues in the control of the tongue.

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